Cosmic string-seeded structure formation

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We describe the results of high-resolution numerical simulations of string-induced structure formation in open universes and those with a non-zero cosmological constant. For models with $\Gamma = \Omega h = 0.1$–0.2 and a cold dark matter background, we show that the linear density fluctuation power spectrum has both an amplitude at $8h^{-1}$Mpc, σ_8, and an overall shape which are consistent within uncertainties with those currently inferred from galaxy surveys. The cosmic string scenario with hot dark matter requires a strongly scale-dependent bias in order to agree with observations.

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A. Introduction—In this Letter we describe new results from an investigation of cosmic string-seeded structure formation in hot and cold dark matter models. The cosmic string scenario [1] predated inflation as a realistic structure formation model, but it has proved computationally much more challenging to make robust predictions with which to confront observation. The present paper relies on high resolution numerical simulations of a cosmic string network [2] with a dynamic range extending from before the matter-radiation transition through to deep in the matter era (developing on previous work [3]). We calculate the linear power spectrum of density perturbations $\mathcal{P}(k)$ induced by the strings in flat models with and without a cosmological constant, and we then extrapolate to open cosmologies. This work represents a considerable quantitative advance by incorporating important aspects of the relevant physics not included in previous treatments.

In the first instance, we consider density perturbations about a flat Friedmann-Robertson-Walker (FRW) model with a cosmological constant Λ and which are causally sourced by an evolving string network with energy-momentum tensor $\Theta_{\alpha\beta}(\mathbf{x},\eta)$. In the synchronous gauge, the linear evolution equations of the radiation and cold dark matter (CDM) perturbations, $\delta_{\rm r}$ and $\delta_{\rm c}$ respectively, are given by (modified from [4])

$$\ddot{\delta}_{\rm c} + \frac{\dot{a}}{a}\dot{\delta}_{\rm c} - \frac{3}{2}\Big(\frac{\dot{a}}{a}\Big)^2 \left(\frac{a\delta_{\rm c} + 2a_{\rm eq}\delta_{\rm r}}{a + a_{\rm eq} + \frac{\Omega_\Lambda}{\Omega_{\rm c}}\frac{a^4}{a_0^3}}\right) = 4\pi G\Theta_+, \quad (1)$$

$$\ddot{\delta}_{\rm r} - \frac{1}{3}\nabla^2\delta_{\rm r} - \frac{4}{3}\ddot{\delta}_{\rm c} = 0, \quad (2)$$

where $\Theta_+ = \Theta_{00} + \Theta_{ii}$, a is the scale factor, the subscript "eq" denotes the epoch of radiation-matter density equality, "0" denotes the epoch today, a dot represents a derivative with respect to conformal time η, $\Omega_{\rm c} = 8\pi G\rho_{\rm c0}/3H_0^2$ and $\Omega_\Lambda = \Lambda/3H_0^2$. It proves useful to split these linear perturbations into initial (I) and subsequent (S) parts [4], $\delta_N(\mathbf{x},\eta) = \delta_N^{\rm I}(\mathbf{x},\eta) + \delta_N^{\rm S}(\mathbf{x},\eta)$, where $N = {\rm c,r}$. The initial perturbations $\delta^{\rm I}(\mathbf{x},\eta)$ depend on the string configuration at some early time $\eta_{\rm i}$, because the formation of strings creates underdensities in the initially homogeneous background out of which they are carved. The subsequent perturbations $\delta^{\rm S}(\mathbf{x},\eta)$ are those which are generated actively by the strings themselves for $\eta > \eta_{\rm i}$. Because strings induce isocurvature perturbations, $\delta^{\rm I}(\mathbf{x},\eta)$ must compensate $\delta^{\rm S}(\mathbf{x},\eta)$ on comoving scales $|\mathbf{x}-\mathbf{x}'| > \eta$ to prevent acausal fluctuation growth on superhorizon scales.

The system of equations (1,2) can be solved for the subsequent perturbations $\delta^{\rm S}(\mathbf{x},\eta)$, with initial conditions $\delta_{\rm c}^{\rm S} = \delta_{\rm r}^{\rm S} = 0$ and $\dot{\delta}_{\rm c}^{\rm S} = \dot{\delta}_{\rm r}^{\rm S} = 0$ at $\eta = \eta_{\rm i}$, by using a discretized version of the integral equation with Green's functions:

$$\delta_N^{\rm S}(\mathbf{x},\eta) = 4\pi G\int_{\eta_{\rm i}}^{\eta} d\eta' \int d^3x'\, \mathcal{G}_N(X;\eta,\eta')\Theta_+(\mathbf{x}',\eta'), \quad (3)$$

where $X = |\mathbf{x}-\mathbf{x}'|$. The Green's functions in Fourier space can be calculated numerically by solving the homogeneous version of (1,2) with initial conditions at $\eta = \eta'$: $\dot{\widetilde{\mathcal{G}}}_{\rm c} = 3\dot{\widetilde{\mathcal{G}}}_{\rm r}/4 = 1$ and $\widetilde{\mathcal{G}}_{\rm c} = \widetilde{\mathcal{G}}_{\rm r} = 0$ ($\widetilde{\mathcal{G}}_N = 0$ for $\eta < \eta'$).

The subsequent perturbations $\delta^{\rm S}(\mathbf{x},\eta)$ are dynamically sourced by moving local strings with spacetime trajectories we can represent as $x_{\rm s}^\mu = (\eta, \mathbf{x}_{\rm s}(\sigma,\eta))$, where σ is a spacelike parameter labelling points along the string (a prime represents a derivative with respect to σ). The stress energy tensor of the string source is then given by [1]

$$\Theta_{\mu\nu}(\mathbf{x},\eta) = \mu \int d\sigma(\epsilon\dot{x}_{\rm s}^\mu\dot{x}_{\rm s}^\nu - \epsilon^{-1}{x'}_{\rm s}^{\mu}{x'}_{\rm s}^{\nu})\delta^3(\mathbf{x}-\mathbf{x}_{\rm s}), \quad (4)$$

where μ is the string linear energy density, $\epsilon = [{\mathbf{x}'_{\rm s}}^2/(1-\dot{\mathbf{x}}_{\rm s}^2)]^{1/2}$, and we have also assumed that $\dot{\mathbf{x}}_{\rm s}\cdot{\mathbf{x}_{\rm s}}' = 0$. In this case, it is straightforward to compute Θ_+ in (1) as

$$\Theta_+(\mathbf{x},\eta) = \Theta_{00} + \Theta_{ii} = 2\mu\int d\sigma\epsilon\dot{\mathbf{x}}_{\rm s}^2\delta^3(\mathbf{x}-\mathbf{x}_{\rm s}). \quad (5)$$

The stress energy $\Theta_{\mu\nu}$ was calculated directly from high resolution string network simulations [2]. Dynamical ranges exceeding 100 in conformal time (redshifts up to 1000) were achievable because of a 'point-joining' algorithm maintaining fixed comoving resolution [5] and parallelization.

B. Approximation schemes—It is a very substantial numerical challenge to evolve the initial and subsequent

perturbations induced by cosmic strings such that they accurately cancel on superhorizon scales by the present day η_0. For the large dynamic range required for the present study, we have by necessity adopted the 'compensation factor approximation' suggested in a semi-analytic context in ref. [6]. To implement this, we accurately evolved the long string network numerically—the dominant active source term—and then multiplied the Fourier transform of the resulting stress energy $\widetilde{\Theta}_+(k,\eta)$ by a cut-off function $\widetilde{F}(k,\eta) = [1+(k_c/k)^2]^{-1}$. This results in the correct k^4 fall-off in the power spectrum at large wavelengths above the compensation scale $k_c^{-1} \sim \eta$. The efficacy of this approximation has been demonstrated by studying multifluid compensation backreaction effects in ref. [7]. For the present study we have adopted the analytic fit for $k_c(\eta)$ presented in ref. [7], which smoothly interpolates from $k_c = \sqrt{6}\eta^{-1}$ in the radiation era to $k_c = \sqrt{18}\eta^{-1}$ in the matter era. The quantitative implementation of compensation is a subtle issue and a key uncertainty in all work to date on gauged cosmic strings. We note, however, that our results are relatively insensitive to the choice of k_c, especially in open and Λ-models (e.g. a large factor of 2 increase in k_c causes only about a 20% decrease in the power spectrum at $k \approx 0.15h^{-1}$Mpc for a flat $\Omega_\Lambda = 0.8$ model).

In order to study the formation of structures with cosmic strings in hot dark matter (HDM) models, we use a reasonably accurate alternative to much more elaborate calculations using the collisionless Boltzmann equation. We simply multiply the Fourier transform of the string source term $\widetilde{\Theta}_+(k,\eta)$ by a damping factor $\widetilde{G}(k,\eta) = \left[1+(0.435kD(\eta))^{2.03}\right]^{-4.43}$ [8]. Here, $D(\eta)$ is the comoving damping length that a neutrino with velocity T_ν/m_ν can travel from the time η onwards. The factor $\widetilde{G}(k,\eta)$ is a fit to numerical calculations of the transfer function of a Fermi-Dirac distribution of non-relativistic neutrinos and accounts for the damping of small-scale perturbations due to neutrino free-streaming [8]. We calculated $D(\eta)$ numerically and found an excellent fit to our results for $T_{\nu 0} = 1.6914\times10^{-13}$ GeV and $m_\nu = 91.5\,\Omega h^2$ eV with $D(\eta) = \frac{1}{20}\log\left[(5\eta_{\rm eq}+\eta)/\eta\right]$.

The other key difficulty confronting defect simulations is their limited dynamic range. At any one time, an evolving string network sources significant power over a lengthscale range which exceeds an order of magnitude. However, we can employ a semi-analytic model to compensate for this missing power [6], which proves to be fairly accurate in the scaling regimes away from the matter-radiation transition. The procedure is essentially to square the expression (3) in Fourier space to obtain the power spectrum $\mathcal{P}(k)$. This becomes a Green's function integral over the unequal time correlators $\langle\Theta_+(\mathbf{k},\eta)\Theta_+(-\mathbf{k},\eta')\rangle$, which are subsumed in a scale-invariant 'string structure function' $\mathcal{F}(\mathbf{k},\eta)$, that is, the power spectrum is given by [6]:

$$\mathcal{P}(k) = 16\pi^2G^2\mu^2\int_{\eta_i}^{\eta_0}|\mathcal{G}_N(k;\eta_0,\eta)|^2\mathcal{F}(k,\eta)d\eta\,. \qquad (6)$$

In practice, we obtained the structure function $\mathcal{F}(\mathbf{k},\eta)$ phenomenologically by fitting its shape and amplitude to simulations of limited dynamic range deep in the matter and radiation eras. We were then able to use an interpolation based on the actual string density during the transition era to provide a good fit to the simulation power spectrum for any given dynamic range $\eta_i \to \eta$.

C. Open and Λ-cosmologies—The possibility that the universe is open or has a cosmological constant is now favoured by a number of oberservations, so it is natural to explore the string-induced spectrum in these two regimes. For the Λ-models $\Omega_c + \Omega_\Lambda = 1$, we have taken the weak dependence for the COBE normalisation of $G\mu \propto \Omega_c^{-0.05}$ suggested in ref. [9] for (3,6). For the open models $\Omega_c < 1$ and $\Lambda = 0$, we simply rescale the simulated spectrum from a flat universe with $\Lambda = 0$ in the following way (adapted from [9]):

$$S(k,h,\Omega_c) = S(k,1,1)\cdot\Omega_c^2h^4\cdot f^2(\Omega_c)\cdot g^2(\Omega_c), \qquad (7)$$

where k is in units of $\Omega_c h^2\,\mathrm{Mpc}^{-1}$ and $f(\Omega_c) = \Omega_c^{-0.3}$ reflects the COBE normalisation of $G\mu$ [9]. The last factor, $g(\Omega_c) = 2.5\Omega_c/(1+\Omega_c/2+\Omega_c^{4/7})$, gives the total suppression of linear growth for density perturbations in an open universe relative to an $\Omega_c = 1$ and $\Omega_\Lambda = 0$ universe [10]. Finally, the second factor $\Omega_c^2h^4$ in (7) arises naturally from the normalization of the Green's functions in (1,2). We have verified that a similar analytic rescaling from an $\Omega_c = 1, \Omega_\Lambda = 0$ model to $\Omega_c + \Omega_\Lambda = 1$ models agrees very accurately with the solutions of (1,2) obtained from simulations. This also helps to justify the extrapolation still required for open models, although some uncertainty remains concerning the COBE normalization.

D. Results and discussion—In figure 1 we plot the Cosmic Microwave Background (CMB) normalized (i.e. $G\mu_6 = G\mu\times10^6 = 1.7$ [11]) linear power spectrum induced by cosmic strings in an $\Omega_c = 1$ CDM cosmology with $h = 0.7$. The central set of numerical points was sourced by string network simulations beginning at $\eta = 0.4\eta_{\rm eq}$ which were continued for 1318 expansion times (from redshift $z_i \approx 31700$ to $z_f \approx 23$). String simulations were always ended before the horizon grew to half the simulation box-size. These had a string sampling resolution at least four times higher than the structure formation grids, which had up to 256^3 points with overall physical scales ranging from 4–100h^{-1}Mpc. Given the dynamic range limitations, we have also plotted the semi-analytic fit (6) over the full range of wavenumbers, illustrating the good agreement with our numerical results. This was also apparent in HDM simulations, so we have considerable confidence that the semi-analytic model provides a good approximation to the shape and amplitude of the string simulation power spectrum. These results

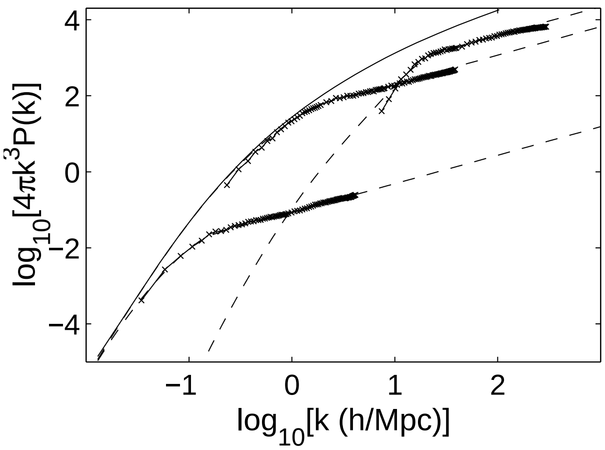

FIG. 1. Comparison of the string-induced simulation power spectrum and the semi-analytic fit (6). The top-right, central, and bottom-left solid lines with crosses are the simulation results in the deep radiation, transition, and deep matter eras respectively. The dashed lines are the semi-analytic fits corresponding to the same dynamical ranges of the simulations. The solid line is the semi-analytic model over the full dynamic range from $\eta_{\rm i} = 0$ to today.

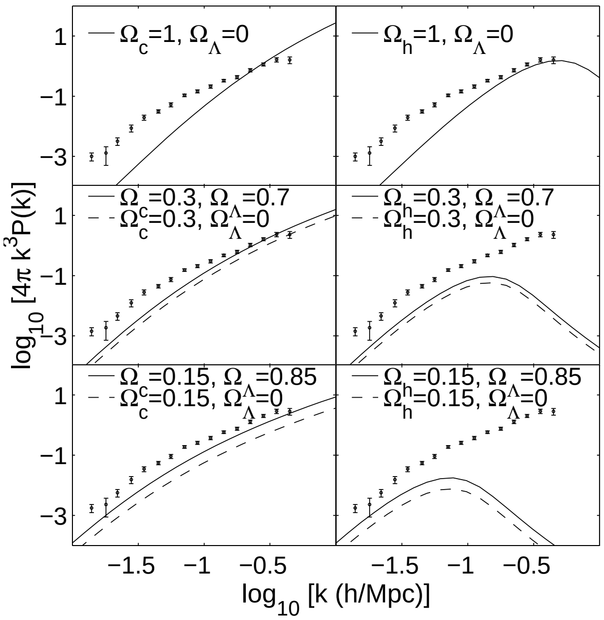

FIG. 2. The CMB normalized linear spectra induced by cosmic strings in CDM (left) and HDM (right) models for different background cosmologies. The solid lines are the flat Λ-models; the dashed lines are the open models. Here, we use $h = 0.7$ and the data points with error bars are the linear spectrum reconstructed from observations [13].

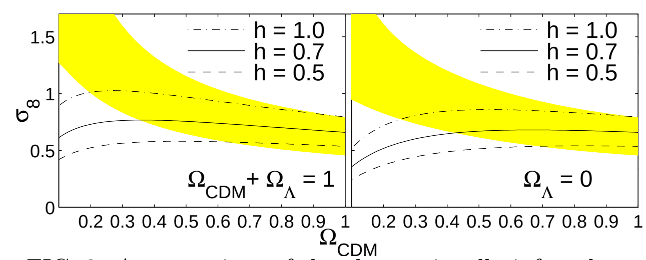

FIG. 3. A comparison of the observationally inferred mass fluctuation $\sigma_{8({\rm obs})}$ with that induced by cosmic strings in our simulations, $\sigma_{8({\rm sim})}$. $\sigma_{8({\rm obs})}$ is shown as the shaded area [15], while $\sigma_{8({\rm sim})}$ is plotted as dot-dashed ($h = 1.0$), solid ($h = 0.7$) and dashed($h = 0.5$) lines.

are also qualitatively consistent with the semi-analytic results of ref. [6] and also with unpublished matter era simulations [12].

In figure 2, we make a comparison between our CDM and HDM string power spectra and the observational results inferred from galaxy surveys [13], in each of five different background cosmologies: (I) $\Omega_{\rm c,h} = 1$, $\Omega_\Lambda = 0$, (II) $\Omega_{\rm c,h} = 0.3$, $\Omega_\Lambda = 0.7$, (III) $\Omega_{\rm c,h} = 0.3$, $\Omega_\Lambda = 0$, (IV) $\Omega_{\rm c,h} = 0.15$, $\Omega_\Lambda = 0.85$, and (V) $\Omega_{\rm c,h} = 0.15$, $\Omega_\Lambda = 0$.

Consider first the $\Omega_{\rm c} = 1$ CDM model. We calculated the standard deviation of density perturbations σ_8 by convolving with a spherical window of radius $8\,h^{-1}\,$Mpc to find $\sigma_{8({\rm sim})}(h = 0.5) = 0.32G\mu_6$, $\sigma_{8({\rm sim})}(h = 0.7) = 0.39G\mu_6$ and $\sigma_{8({\rm sim})}(h = 1.0) = 0.47G\mu_6$. A comparison with the observational data points shows that strings appear to induce an excess of small-scale power and a shortage of large-scale power, that is, the $\Omega_{\rm c} = 1$ string model requires a significant scale-dependent bias. This is not necessarily a fatal flaw on small scales because, as the corresponding HDM spectrum indicates, such excess power can be readily eliminated in a mixed dark matter model. However, the problem is less tractable on large scales where biases up to $\sigma_{100({\rm obs})}/\sigma_{100({\rm sim})} \approx 3.9$ around $100h^{-1}$Mpc might be inferred from the data points (using $G\mu_6 = 1.7$ and $h = 0.7$). Should we, therefore, rule out string models on this basis [14]? Although the $\Omega_{\rm c} = 1$ spectrum looks unattractive, there are three important mitigating factors. First, the present observational determination of the power spectrum around $100h^{-1}$Mpc is very uncertain. Secondly, the immediate nonlinearity of string wakes means that strong biasing mechanisms might operate on large scales. Finally, unlike inflation, defect models have never been wedded to an $\Omega = 1$ cosmology.

[b] We can observe from figure 2, that for open or Λ-cosmologies with $\Omega_{\rm c} \approx 0.1$–$0.3$, the string + CDM power spectrum is much more encouraging. We find that the bias on large scales is always reasonably close to unity and, overall, it is much less scale-dependent. For example, over the full range of lengthscales in model IV ($\Omega_{\rm c} = 0.15$, $\Omega_\Lambda = 0.85$), the relative bias remains $\sigma_{100({\rm obs})}/\sigma_{100({\rm sim})} \approx 1.4 \pm 0.2$ at $100h^{-1}$Mpc. In figure 3, the value of $\sigma_{8({\rm sim})}$ induced in our simulations with the CMB normalized $G\mu_6$ [9,11] is compared with the observationally inferred $\sigma_{8({\rm obs})}$ [15] for the full gamut of open and Λ-models. We can see from figure 3, that $\sigma_{8({\rm obs})}/\sigma_{8({\rm sim})}(\Omega_{\rm c}{=}1) \approx 0.79 \pm 0.21,\ 0.95 \pm 0.25,\ 1.17 \pm 0.31$ for $h = 1.0, 0.7, 0.5$ respectively. When $h = 0.7$,

$\sigma_{8(\mathrm{sim})}$ matches $\sigma_{8(\mathrm{obs})}$ within the uncertainties for flat Λ-models when $\Omega_c \gtrsim 0.35$ and for open models when $\Omega_c \gtrsim 0.4$, while for both cases the ratio $\sigma_{8(\mathrm{obs})}/\sigma_{8(\mathrm{sim})} \lesssim 2$ for all $\Omega_c \gtrsim 0.1$. Combining these results with an analysis similar to figure 2, we found that the best string models lie in the range $\Gamma = \Omega h = 0.1$–0.2, producing both an acceptable $\sigma_{8(\mathrm{sim})}$ and power spectrum shape. Hence, an open or Λ-cosmology in the context of string + CDM model certainly merits a more detailed nonlinear study. These conclusions are in qualitative agreement with semi-analytic results [9,16] and those based on a phenomenological string model [17].

As for the HDM results, the comparison with observation seems to require a strongly scale-dependent bias for any choice of the cosmological parameters (models I–V). However, the lack of small scale power may be partially overcome if baryons are properly included in the analysis. Further investigation using a hydrodynamical code will be required to determine whether galaxies can form early enough.

A key feature of all these string-induced power spectra is the influence of the slow relaxation to the matter era string density from the much higher radiation string density, which has an effective structure function $\mathcal{F}(k,\eta)$ in (6) with approximately 2.5 times more power than the matter era version. Even by recombination in an $\Omega_c = 1$ cosmology, the string density is more than twice its asymptotic matter era value to which we normalize on COBE scales. This implies that the string model provides higher than expected large-scale power around $100h^{-1}$Mpc and below. Interestingly, this can also be expected to produce a significant Doppler-like peak on small angle CMB scales, an effect noted in ref. [11] but not observed because only matter era strings were employed. Recent work in ref. [17] confirms that such Doppler-like features can result from significant nonscaling effects during the transition era.

Finally, we comment on the fact that the key uncertainties affecting these calculations primarily influence the amplitude of the string power spectrum, rather than its overall shape which appears to be a more robust feature. These uncertainties mainly result from the compensation approximation (mentioned previously), the COBE normalization of the string energy density [11,18], the analytic approximation to the Green's functions, and systematic errors [19]. Combining our best estimates of these uncertainties gives an approximate factor of 2 uncertainty in the power spectrum amplitude for $\Omega_c = 1$ and Λ-models. The extrapolations required for open models with $\Gamma \approx 0.15$ increase this uncertainty to at least a factor of 3 overall, but we will discuss this at length elsewhere [19].

E. Conclusion—We have described the results of high-resolution numerical simulations of structure formation seeded by a cosmic string network with a large dynamical range taking into account, for the first time, modifications due to the radiation-matter transition. Our results show that for $\Gamma = \Omega h = 0.1$–0.2 both σ_8 and the power spectrum shape of cosmic string-induced CDM fluctuations agree satisfactorily with observations. In particular, the generalization to open or Λ-models tends to remove the excess small-scale power found in cosmic string models with $\Omega_c = 1$ and $\Omega_\Lambda = 0$, while also bolstering the large-scale power. The HDM power spectrum requires a strongly scale-dependent bias either on small or large scales, but we note that a high baryon fraction may help to increase small-scale power. We conclude that the picture which emerges for particular cosmic string models seems encouraging and certainly deserves further study [19].

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